Radiometer equ.

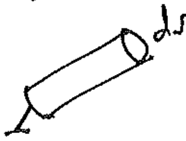
- Really about SNR: my telescope / observations has these characteristics, can I see this source?

End result: $\dfrac{S}{N} = \dfrac{T_{source}}{T_{sys}} \sqrt{\tau \Delta\nu}$

- step back for a minute: what's up w/ all the temps??
- Background:
    - This is the Rayleigh-Jeans limit of the blackbody equation:

$$I_\nu \approx \frac{2kT_{src}}{\lambda^2} \qquad [I_\nu] = \frac{\text{erg}}{\text{s}\cdot\text{cm}^2\cdot\text{Hz}\cdot\text{ster}}$$

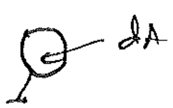
- what's the flux density received by antenna? Int. over Ω subtended by antenna



$$\int I_\nu \, d\Omega = S_\nu \qquad [S_\nu] : \frac{\text{erg}}{\text{s}\cdot\text{cm}^2\cdot\text{Hz}}$$

$$1 \text{ Jy} = 10^{-23} \frac{\text{erg}}{\text{s}\cdot\text{cm}^2\cdot\text{Hz}}$$

- what's the actual power you receive? Int. over antenna's area



$$\int S_\nu \, dA = P_\nu \qquad [P_\nu] = \frac{\text{erg}}{\text{s}\cdot\text{Hz}}$$

- Antenna theorem:
    - we basically just did this:

$$(I_\nu)\cdot\Omega\cdot A_e \approx \frac{2kT}{\lambda^2}\,\Omega\cdot A_e$$

- Antenna theorem: $\lambda^2 = \Omega \cdot A_e$

So, $\boxed{P_\nu = 2kT_{src}}$

- Now, convert between T_{src} and J_y (intrinsic property of src.):

$$P_\nu = 2kT_{src}$$
$$\downarrow$$
$$S_\nu A_e = 2kT_{src}$$
$$T_{src} = \left(\frac{A_e}{2k}\right) S_\nu$$

"forward gain" performance param. of antenna, in $\frac{K}{Jy}$ (or $\frac{Jy}{K}$, if flipped)

- Will mention effective area (A_e) later.

* we've converted from $T \to Jy$ ... cool. We see that T_{src} is just a measure of the brightness of the src. we're looking at.

Now: back to eqn: $\frac{S}{N} = \frac{T_{src}}{T_{sys}} \sqrt{\tau \, \Delta\nu}$

T_{sys}? $\quad T_{sys} = T_{sky} + T_{rx}$ — Power generated by everything you don't want to look @ on the sky, & by the electrical components in your telescope

T_{sky}: T_{atm}, $T_{bg\ src}$ ...

T_{rx}: thermal (or Johnson noise in mixer, amplifier, etc.

Back to A_{eff}: it cuts down on signal that comes in (b/c of dish properties), so you have to amplify the signal!

- If you're Rx-noise dominated, A_{eff} matters, b/c you amp rx noise
- If you're atm-noise limited (say, mm-wave on humid day), then A_{eff} doesn't matter b/c it cuts down on sky noise, which then just gets amped back up.

Now: how do we detect a signal?

Typically $T_{src} < T_{sys}$

- Beat down the noise!

Need $T_{src} > T_{rms}$ $\longrightarrow$ $\underset{\uparrow \text{ obs-dep.}}{T_{rms}} = \frac{T_{sys}}{\sqrt{N}}$ stand.

w/ telescope: $N = BW \cdot time = \overset{(Hz)}{\Delta\nu} \cdot \overset{(s)}{\tau}$

So: $T_{rms} = \frac{T_{sys}}{\sqrt{N}} = \frac{T_{sys}}{\sqrt{\Delta\nu\,\tau}}$

Arrive @ $\frac{S}{N} = \frac{T_{src}}{\frac{T_{sys}}{\sqrt{\Delta\nu\tau}}} = \boxed{\frac{T_{src}}{T_{sys}}\sqrt{\Delta\nu\tau}}$!

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① Final note: recast in terms of S_ν, not T.

Rewrite variables & introduce $SEFD = \frac{2k\,T_{sys}}{A_e}$ $[SEFD] = Jy$ fund. property of antenna

$S_{\nu,rms} = \frac{SEFD}{\sqrt{\tau\Delta\nu}}$

② Extension to interferometry (watch "intro to interf. video first"):

$$S_{\nu,rms} = \frac{SEFD}{\sqrt{\underbrace{\frac{N(N-1)}{2}}_{\#\text{ of pairs of antennas}} \cdot \tau \cdot 2\Delta\nu}} = \frac{SEFD}{\sqrt{N(N-1)\cdot\tau\Delta\nu}}$$

b/c measuring amp. + phase (or b/c you have two ants making measurement)